

# **Income ratio and the Risk-sharing Structure of Optimal Contracts: The break-even Theory of *Muḍārabah***

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*Using basic postulates of the standard portfolio analysis, it is shown that the risk-sharing ratio of a two-party (manager/financier) contract cannot be viewed independently of its income ratio. A negative relationship tends to exist between the income ratio of any one party, and his risk-sharing ratio. Thus, a downwards sloping optimal contracts curve (OCC) has been established. This property reflects the market competitive forces of utility maximizing parties within an informational efficient environment.*

*Interestingly, the negative slope of the OCC justifies the emergence of the muḍārabah contract as a break-even point where the two competitive market forces are balanced. It is the point where the OCC intersects the profit-sharing line, and where risk and income prove to be shared fairly from two perspectives: first, the risk of muḍārabah is shared between its two parties in exactly the same ratio as income. Second, the muḍārabah contract implies equal income shares (income ratio =  $\frac{1}{2}$ ) when parties are equally risk-averse. Cases of unequally risk-averse parties are examined, but on average muḍārabah financing under pure competitive conditions seems to be most conducive to sharing ratios in a close region of one half.*

*This finding provides a meaningful interpretation to the concept of fairness in muḍārabah. In the current literature, the ethical appeal of muḍārabah, as against fixed interest financing, tends to be emphasized with reference to the risk-sharing property that exists in the former but not in the latter. Yet, little attention is paid to possible differences in the income ratio. One important implication is that the muḍārabah profit-sharing ratio cannot be maneuvered freely and independently as a tool of monetary policy without adversely affecting its ethical appeal. Such maneuvering is also likely to have an adverse effect on the supportive risk sharing structure of muḍārabah.*

## **1. Introduction:**

Profit-sharing through *muḍārabah* financing is believed to be the genuine financing alternative to the forbidden interest rate system in Islamic economics.

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Theoretical interest in the current dialogue about optimal financial contracts from Islamic perspective can be traced back to an initial work by Khan (1985), who established a Pareto-optimal theorem for *muḍārabah* when compared to fixed interest rate financing under conditions of informational symmetry. Khan theorem, however, suffered from a fundamental weakness as it implied *muḍārib*'s risk neutrality, although the author argued otherwise (Tag el-Din, 1991). Nonetheless, Khan's theorem was a landmark in the literature on Islamic economics as it brought to a sharp focus the issue of informational asymmetry in the *muḍārabah* contract and its consequent moral hazard problem. Ever since, increasing interest in the theory of optimal contracts and incentive-compatible systems for profit-sharing systems followed suit, as exemplified by ul-Haque and Mirakhor (1986), Tariquallh Khan (1995), Bashir (1996), and Habib (2001). More recently, various contributions with regard to the moral hazard problem have been made in Iqbal (2001), and Iqbal and Llewellyn (2002) by Abalkhail and Presley (2002) as well as Khalil et al (2002).

The common feature of recent works on incentive-compatible schemes has been to address the principal/agent problem arising from informational asymmetry in the *muḍārabah* contract between financier (*rabb al-māl*) and financee (the *muḍārib*). A fundamental hypothesis taken for granted in these studies is that, had it not been for informational asymmetry, the *muḍārabah* contract would dominate over prefixed return contracts. In other words, the *muḍārabah* contract should dominate over prefixed rate financing within a theoretical model of informational symmetry. This hypothesis, however, has to be thoroughly examined if concerns with incentive-compatible schemes in *muḍārabah* has to make sense.

Little attention seems to have been given to the risk-return optimality properties of *muḍārabah* under informational symmetry, except perhaps for Tag El-Din (1992, 2002) who recommended a risk-return sharing model (RRSM). The latter is a special version of an Edgeworth box where two risk-averse parties (capital provider and manager) are represented within an ex ante informational efficient environment. The RRSM presents the problem of financial choice as one involving three possible options: a pure variable return (through *muḍārabah* profit sharing), a pure risk-free fixed return and a combination of the two. Interestingly, it has been shown that Pareto-optimality is satisfied by the pure variable return and a set of possible combinations with the risk-free fixed return, whereas the pure risk-free return fails to satisfy the Pareto-optimality criterion. Only under inefficient informational conditions would a pure risk-free return arise within the RRSM. It has, thus, been concluded – in line with the theoretical implications of the standard CAPM – that an optimal contract must possess a definite risk-sharing structure. In particular, the *muḍārabah* contract emerges as a special case of an optimal contract where risk is shared in the same ratio as income (Tag El-Din, 2002).

The objective of this paper is to explain the pattern of various possible risk-sharing structures embodied by the given two-party optimal contracts. The idea is

to see how the risk-sharing structure of a typical two party contract is related to the income ratio which underlies financial contracts.

### 1.1 Portfolio Analysis within RRSM

Given the assumption of expected utility maximization, it is interesting to explore the status of the *muḍārabah* contract within an ‘optimal contracts curve’ describing the relationship between income ratio and risk-sharing ratio. Naturally, the query about the optimal relationship between income ratio and risk-sharing ratio would never arise if the parties were risk-neutral. In the latter case, all that matters would be the income ratio whereby each party seeks to maximize his income share subject to his competitive market position. A risk-neutral party would bargain for a maximum possible income ratio regardless of the risk-structure of the contract.

Yet, we are assuming that both parties are risk-averse, in conformity with the standard practice, and hence the risk-sharing structure does matter. Each party is assumed to seek maximum expected utility, in the sense of maximum possible income share at the minimum possible risk share. Incidentally, the assumption of risk-aversion is also implicit in the risk-sharing jurisprudence of *muḍārabah*. Recognition of risk as an undesirable fact of economic life, and that people would normally wish to throw it on others’ shoulders, explains the Islamic concept of justice that risk must be shared fairly between the parties (Tag El-Din, 2002).

Admittedly, there is more to the jurisprudence of *muḍārabah* than what can be derived through the utility maximization assumption. It is possible from a jurist viewpoint that one of the two parties behaves charitably, hence, accepts a very small share, or even no share at all in the *muḍārabah*’s profit. The idea of *ibdaa*’ is the case in point where all profit is donated by the *muḍārib* to the financing party (*rubb al-māl*) (*Al-Mawsu’at Al-Kuwaitiya* (1993), p.172-178). But, altruism cannot be taken as the general rule in the financial market. If the ethical rule in market dealings is ‘fairness’ rather than charitableness, it would be interesting to see how possible it is for expected utility maximization to yield an ethical result.

### 1.2 The Income Ratio

The ‘income ratio’ is used here to denote the division of income between financier and manager in a two party contract, regardless of how risk is shared between them. Although the relationship between the income ratio and the risk-sharing structure of a financier/manager contract is the subject of inquiry, the income ratio can be defined independently of the risk-sharing structure. The income ratio, therefore, denotes any division of expected income between the two parties. It may denote the division of income between a risk-taking entrepreneur (manager) and a risk-free lender (financier), as in the neo-classical model of the firm. Alternatively, it may denote profit-sharing *muḍārabah* where risk is shared between *rubb al-māl* (capital financier) and *muḍārib* (manager). A third possibility

is the division of income between a risk-free salaried manager and a risk-taking financier as in the employer/employee model. Still, two other hybrid possibilities may arise where profit-sharing is combined with either fixed interest or fixed salary. Thus, we have already defined five risk-sharing structures which can be associated with any given income ratio. The idea is to see how such structures are influenced by the given income ratio.

Looked at independently of risk-structures, the income ratio is essentially a distribution index reflecting the relative competitive positions of two market forces - a human resource owner (manager) as against capital owner (saver) - each seeking the largest possible share of the jointly generated income cake. Although it is not an explicitly recognized index, the income ratio is usually taken for granted by contracting parties in competitive environments. In actual practise Islamic banks seem to be using the market interest rate as a proxy of the income ratio to help decide on *muḍārabah* profit-sharing ratios or competitive rates of *murābahah* mark-up.

## 2. Basic Background.

The representation of a security's expected return and risk, respectively, in terms of mean and standard deviation is the seminal idea that has paved the way for the growth of the modern portfolio theory since the early sixties of the last century. The assumptions of the modern portfolio theory are fully detailed by Haugen (1986, pp155-184). Apart from the usual free market competitive conditions, there are two basic assumptions to highlight:

- Informational efficiency: This term is defined in terms of known distribution parameters of investment returns for all financial assets within an uncertain environment. Thus, the subject matter of market information are the return and risk parameters – mean and standard deviation – assumed to be known by all potential participants.
- Attitudinal assumption: utility maximizing investors can choose between different portfolios on the basis of mean and standard deviation. The mean-variance indifference curve is assumed to be upwards sloping and convex from below. The quadratic utility function is adopted as it is often assumed as a basic simplification.

The beginning of modern portfolio theory dates back to 1952 when Harry Markowitz introduced the concept of a mean-variance efficient frontier for a set of investment securities. However, the theory acquired its computational convenience mainly through the 'single index model', introduced in 1963 by William Sharpe. Subsequent theoretical refinements and practical developments led to the formulation of the Capital Assets Pricing Model (CAPM) by Sharpe, Litner, and Mossin. In a nut-shell, the CAPM presents the efficient portfolio as a single combination of risky securities (The market portfolio) augmented by borrowing

and lending along a Capital Market Line. The CAPM continues to maintain its recognition in the standard textbooks of finance, and the advanced computerized packages of financial analysis, notwithstanding the severe theoretical criticisms triggered off by Roll (1976).

Attitude towards risk is the decisive element of the whole exercise. Admittedly, the whole risk-return structure will boil down to pure mathematical tautology unless it is relevant to an economically consistent scale of preference. A typical investor is, thus, assumed to select his efficient portfolio in terms of a convex-downwards risk-return indifference curve. Tobin makes the assertion that for normally distributed returns the convexity property must necessarily hold ((Tobin(1958, 1974)), Feldestien (1969), Borch (1974) ).

The basic implication of the CAPM which seems to contradict Islamic economics is the presentation of capital market lending and borrowing at a fixed risk-free interest rate as an integral part of an efficient portfolio, given that fixed rate lending is not permitted in Islamic jurisprudence. This point has been addressed by Tag El-Din (1991) through his questioning of Tobin's assertion with regard to the downwards-convexity property of investors' risk-return indifference curves. Nonetheless, it has been shown through the RRSM that the same convexity property will still present *muḍārabah* financing as a Pareto-optimal contract.

### 3. The Two Party Model

The basic model assumes two contracting parties, A and B, aiming respectively to provide finance and management for an income-yielding project. All finance is provided by Party A (the capital provider), while all management is provided by Party B (the Manager). Thus, the basic model can be expressed in terms of three random variables X, Y, and Z, where X stands for the project's total income, Y stands for A's share in total income, and Z for B's share. The term 'income' will be used interchangeably with expected 'return'. Henceforth, the return-risk parameters,  $(\theta, \sigma)$ , are defined respectively as the mathematical mean,  $\theta = E(X)$  of the total income variable X, and standard deviation,  $\sigma$ , of the same variable. The information efficiency assumption implies that the parameters  $(\theta, \sigma)$  are known by both parties at the time of contracting.

We have deliberately introduced income-sharing and risk-sharing parameters  $(\alpha, \beta)$ , respectively, to define different possible contracts between the two parties. Hence, any particular contract between A and B becomes directly expressible in terms of specific  $\alpha$  and  $\beta$ , as in the general forms:

- A's income-risk shares:  $(\alpha\theta, \beta\sigma)$ .
- B's income-risk shares:  $((1-\alpha)\theta, (1-\beta)\sigma)$ , [1]

where  $0 < \alpha < 1$  and  $0 \leq \beta \leq 1$ .

The question, therefore, is how to optimally share the given  $(\theta, \sigma)$  parameters between the two parties. For analytical convenience we shall start up from a given market-determined income ratio:

$$\alpha = \alpha_0 \quad [2]$$

Given the above fixed income ratio, each of the two parties will continue to expect the same income share, regardless of the risk-sharing parameter,  $\beta$ . That is,

$$\begin{aligned} E(Y) &= \alpha_0 \theta \\ E(Z) &= (1 - \alpha_0) \theta \quad \text{for all } 1 \geq \beta \geq 0 \end{aligned} \quad [3]$$

Hence, the only way to distinguish between different possible contracts will be through specific values of the risk-sharing parameter,  $\beta$ , as introduced in [1] above. On this basis, the general forms of the income risk-sharing structures for the two parties, A and B, as they are given by the random variables Y and Z, respectively, subject to the condition  $\alpha = \alpha_0$ , can be written as:

$$\begin{aligned} Y &= f(X; \beta) \\ Z &= g(X; \beta), \quad 1 \geq \beta \geq 0 \end{aligned} \quad [4]$$

### 3.1 The Five Possible Contracts:

With reference to [3] and [4] above, five possible risk-sharing schemes can be defined between the two contracting parties. Figure (1) below is a geometrical representation of the return-risk sharing model (RRSM) involving the two Parties A and B on opposite sides of the box as in Edgeworth Box. Party A is represented on the regular return-risk axes, while Party B is represented by the inverted ones. The Profit Sharing Line runs diametrically from the north-eastern corner to the south west corner of the box. All possible contracts between the two parties are shown along the horizontal income ratio line,  $\alpha = \alpha_0$ . Note that the last condition stands for Party A's fixed income share, regardless of his risk-share, while  $(1 - \alpha_0)$  stands, similarly, for Party B's income share. In particular, the following set of contracts are defined from Party A's perspective. By symmetry these can, similarly, be defined from Party B's perspectives:

#### *Partial borrowing Contract:*

Here  $0 < \beta < \alpha_0$  implying that Party A will bear less share in risk than his given income share. It means that Party A's expected share is partly defined as a risk-free fixed interest, 'r', and partly as a share,  $\beta$ , in the total return variable, X. Thus, equations [4] for the two parties above can be represented as :

$$\begin{aligned} Y &= r + \beta X \\ Z &= (1 - \beta) X - r \end{aligned} \quad [5]$$

The property  $0 < \beta < \alpha_0$  follows by combing [3] and [5] above.

*Partial hiring Contract:*

Here  $\alpha_0 < \beta < 1$ , implying that Party A will bear more share in risk than his market share in return. It is the case where management service is acquired by Party A, partly through the hiring of Party B's services at the risk-free fixed salary, 's', and partly as an equity share. Hence:

$$\begin{aligned} Y &= \beta X - s \\ Z &= s + (1 - \beta) X \end{aligned} \quad [6]$$

Again, the property  $\alpha_0 < \beta < 1$  follows by combining [3] and [6] above.

*The Muḍārabah Contract:*

Here,  $\beta = \alpha_0$ , implying that the share of risk born by either Party A or party B is exactly equal to his assumed share in return. Hence:

$$\begin{aligned} Y &= \beta X \\ Z &= (1 - \beta) X \end{aligned} \quad [7]$$

The property  $\beta = \alpha_0$  follows from  $r = s = 0$ . In this case, Party A is *rubb al-māl*, while Party B is the *muḍārib*. Notably, *muḍārabah* is the only contract where a party's share in risk is exactly equal to his share in return.

*The pure Borrowing model:*

Here,  $\beta = 0$ , implying that Party A bears no risk at all. All risk is born by Party B, who borrows at the risk-free rate,  $r$ , from Party A. It is simply represented by:

$$\begin{aligned} Y &= r \\ Z &= X - r \end{aligned} \quad [8]$$

The risk-free interest  $r$  is a fixed expected share  $\alpha_0\theta = E(Y) = r$ , bearing in mind that  $\theta$  is a known number at the time of contracting.

*The Pure Hiring Contract:*

Here,  $\beta = 1$ , implying that all risk is born by Party A, who hires Party B's management services at a risk-free salary 's'. It is represented by:

$$\begin{aligned} Y &= X - s \\ Z &= s \end{aligned} \quad [9]$$

As in the previous case, the risk-free return, 's', is the fixed expected share:

$$(1 - \alpha_0)\theta = E(Z) = s.$$

Note that, the property  $\beta = \alpha_0$  defines the *muḍārabah* contract at the break-even point where the horizontal line  $\alpha = \alpha_0$  cuts the Profit Sharing line. Accordingly, all

points falling to the right of the Profit Sharing Line are of partial borrowing contracts, while those falling to the right are of the partial hiring contracts. Pure borrowing/hiring contracts are defined, respectively, on the left hand boundary,

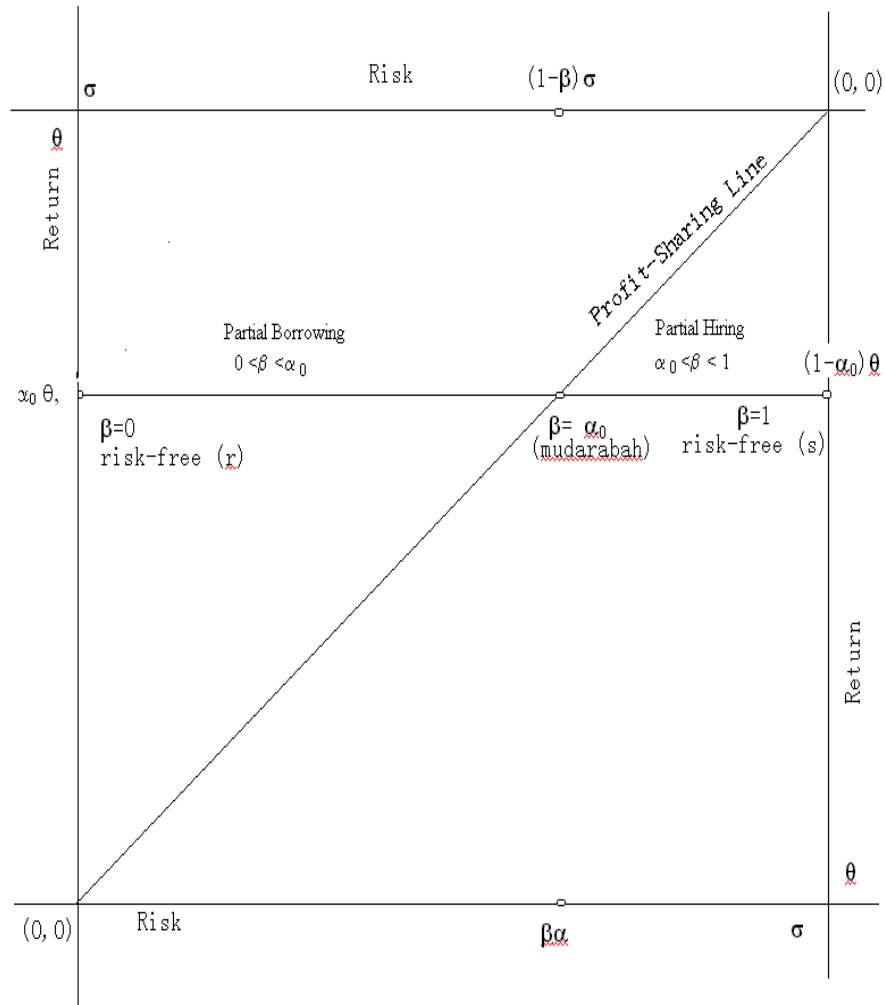
$\beta = 0$ , and the right hand boundary,  $\beta = 1$ .

### 3.2 Pareto-Optimal Financial Structures

The above set of contracts are represented by Figure [1] below for any given fixed income ratio  $\alpha = \alpha_0$ . But the idea is to trace the risk-sharing structures for all possible income sharing ratios which satisfy the Pareto-optimality criterion; that is, where no one party can be made better off without making the other party worse-off. The result will be an optimal contract curve, (OCC) which will make it possible to match any given income ratio,  $\alpha_0$ , with its corresponding risk-sharing ratio. The question is about shape of the OCC which reflects the action of two competitive market forces (owners of a human resource as against owners of financial capital), each seeking to acquire maximum utility of the contract's expected income cake.



Figure 1: The five set of possible contracts



#### 4. The Optimal Contract Curve

To examine the shape of the OCC, we shall adopt a quadratic utility function. The expected utility functions for the two parties A and B, are given respectively, as:

$$\begin{aligned} U_A(\theta_1, \sigma_1) &= a_0 + a_1\theta_1 + a_2(\theta_1^2 + \sigma_1^2), \\ U_B(\theta_2, \sigma_2) &= b_0 + b_1\theta_2 + b_2(\theta_2^2 + \sigma_2^2) \end{aligned} \quad [10]$$

Where  $\theta_1 = \alpha\theta$ ,  $\sigma_1 = \beta\sigma$ , for Party A, while  $\theta_2 = (1-\alpha)\theta$  and  $\sigma_2 = (1-\beta)\sigma$  for Party B. Hence,  $\theta = \theta_1 + \theta_2$  and  $\sigma = \sigma_1 + \sigma_2$ . Note that  $a_0$  and  $b_0$  are any arbitrary constants. To guarantee a positive marginal utility of return for both parties, we must impose the two conditions:

$$a_1 + 2a_2\theta_1 > 0, \text{ and } b_1 + 2b_2\theta_2 > 0, \quad [11]$$

defined over a relevant range of the return parameter. The parameters:

$$a_2 = \partial^2 U_A / \partial \sigma_1^2 < 0, \text{ and } b_2 = \partial^2 U_B / \partial \sigma_2^2 < 0, \quad [12]$$

are the respective measures of risk aversion for the two parties. For example, Party A will be less risk-averse than Party B only if  $a_2 > b_2$ .

The OCC for the two parties, is defined in terms of sharing values  $\alpha$ ,  $\beta$  which maximize a given party's utility function subject to any fixed level of the other's utility. The constrained utility function can be defined symmetrically for either of the two parties without affecting the result. For Party A it is:

$$U^* = U_A(\theta_1, \sigma_1) + \lambda [U_B(\theta_2, \sigma_2) - U^{(0)}_B]$$

where  $\lambda$  is the Lagrange multiplier, and  $U^{(0)}_B$  is an arbitrarily fixed level of Party B's utility function. The first order conditions of constrained maximization are:  $(\theta_1, \sigma_1)$

$$\partial U^* / \partial \theta_1 = \partial U_A / \partial \theta_1 - \lambda \partial U_B / \partial \theta_2 = 0$$

$$\partial U^* / \partial \sigma_1 = \partial U_A / \partial \sigma_1 - \lambda \partial U_B / \partial \sigma_2 = 0$$

$$\partial U^* / \partial \lambda = U_B(\theta_2, \sigma_2) - U^{(0)}_B = 0$$

Then, the condition of Pareto optimality is given as:

$$(\partial U_A / \partial \theta_1) / (\partial U_A / \partial \sigma_1) = (\partial U_B / \partial \theta_2) / (\partial U_B / \partial \sigma_2) \quad [13]$$

Or, alternatively, through the quadratic utility functions, as:

$$(a_1 + 2a_2\theta_1) / 2a_2\sigma_1 = (b_1 + 2b_2\theta_2) / 2b_2\sigma_2 \quad [14]$$

Then, to focus primarily on risk-aversion rates we may simplify the above formula by letting  $a_1 = b_1 = c$ , leading to:

$$(c + 2a_2\theta_1) / 2a_2\sigma_1 = (c + 2b_2\theta_2) / 2b_2\sigma_2 \quad [15]$$

## 5. Break-Even Theory of *Muḍārabah*:

### 5.1 Case of equally risk-averse parties:

The break-even theory of *muḍārabah* within a competitive market can be described under various patterns of risk-aversion rates. The case of equally risk-averse parties implies  $a_2 = b_2 = d$  is the simplest one. On this basis, it is possible to establish the following properties:

1. There exists a linear relationship between the income ratio  $\alpha$  and the risk-sharing parameter  $\beta$  of the form:

$$\alpha = m + w\beta \quad [16]$$

where  $m = -c/2d\theta$ , and  $w = (c + d\theta)/d\theta$ . From [11] and [12] above, it follows that  $w < 0$ . Hence,  $\alpha$  and  $\beta$  are negatively related through a downwards sloping optimal contracts curve (OCC), as shown in figure (2). In principle, the OCC traces the tangential points of the two parties' indifference curves, but, to avoid complicating the figure, the tangential points are omitted. The linear OCC shows that: the higher the income ratio for any party, the smaller the risk-share of that party. Conversely, the smaller the income ratio of any party, the greater is his risk-share.

This property reflects the action of competitive market forces by the two utility maximizing agents, each seeking a higher income share at a lower risk share. Correspondingly, the stronger party is able to achieve both objectives: more return share and less risk share. The weaker party of the market will be obliged to accept a smaller income share and a bigger share in risk.

2. That the boundary contracts pure borrowing and pure hiring contracts are not represented on the OCC. This follows from [16] where it can be shown that:

$$0 < \beta < 1, \text{ for all } 0 \leq \alpha \leq 1,$$

Thus, the OCC accommodates only the contracts with definite risk-sharing structures (partial borrowing, partial hiring, and *muḍārabah*) as defined above.

3. That the immediate effect of a negatively sloping OCC is to yield the *muḍārabah* contract at the break-even point where the relative competitive market forces are fairly balanced. In the current case of equally risk-averse parties, the *muḍārabah* contracts is located at the centre of gravity ( $\frac{1}{2}, \frac{1}{2}$ ) of the RRSM, where the contracting curve intersects the positively sloping profit-sharing line. The optimal profit-sharing ratio for *muḍārabah* financing turns out to be  $\alpha = \frac{1}{2}$ , giving equal income shares for the two parties.

Two ethical distribution properties seem to be associated with *muḍārabah* financing: *fair risk-sharing* and *fair income ratio*. The last point is particularly interesting, as it does not seem to be recognized in the current literature.

Next, it is interesting to see how the alternative situation of ‘unequal’ risk-aversion may affect the above property of equal income shares in *muḍārabah* financing. Any change in the OCC position should affect the profit-sharing break-even point for *muḍārabah* financing.

## 5.2 Case of Unequally Risk Averse Parties

It is easy to see that if one party is risk-averse and the other is risk-neutral, then the OCC will totally coincide with any one of the two boundaries of the model, leading to either *pure borrowing* model or *pure hiring* model. For example if Party B (the manager) is risk neutral while Party A (the financier) is risk-averse, the OCC will coincide with the pure fixed interest rate boundary for Party A, leading to a pure borrowing model.

By symmetry, less restrictive results can be expected for the less restrictive case of *relative risk neutrality* where one party is closer to risk neutrality (less risk-averse) than the other. In the latter case, the OCC will come closer to a boundary rather than coincide with it. In general, we should expect the OCC to lie closer to the fixed return boundary for one party, the closer to risk-neutrality is the other party. Using the central point of the model (  $\frac{1}{2}$ ,  $\frac{1}{2}$  ) as bench mark, if the OCC is positioned to the left of (  $\frac{1}{2}$ ,  $\frac{1}{2}$  ), then it must be closer to the fixed interest boundary. If it is positioned to the right of (  $\frac{1}{2}$ ,  $\frac{1}{2}$  ), then it must be closer to the fixed salary boundary.

Then, to account for different attitudes towards risk, we shall define the risk attitudinal differential (RAD) in terms of the risk aversion parameters  $a_2$ ,  $b_2$  with reference to [12], as :

$$\text{RAD} = a_2 - b_2 \quad [17]$$

Notably,  $(a_2 - b_2) > 0$  implies that Party A is closer to risk neutrality than Party B, while  $(a_2 - b_2) < 0$  implies that Party B is the closer to risk

neutrality. We have just seen that in case of  $\text{RAD} = 0$ , the contract curve passes through the central point of the model (  $\frac{1}{2}$ ,  $\frac{1}{2}$  ). It will be interesting to see how the RAD affects the relative positioning of the optimal contracting curve.

Hence, we may rewrite the Pareto-optimality condition of equation [14]

$$(a_1 + 2a_2 \alpha \theta) / (b_1 + 2b_2 (1-\alpha)\theta) = a_2 \beta / b_2 (1-\beta) \quad [18]$$

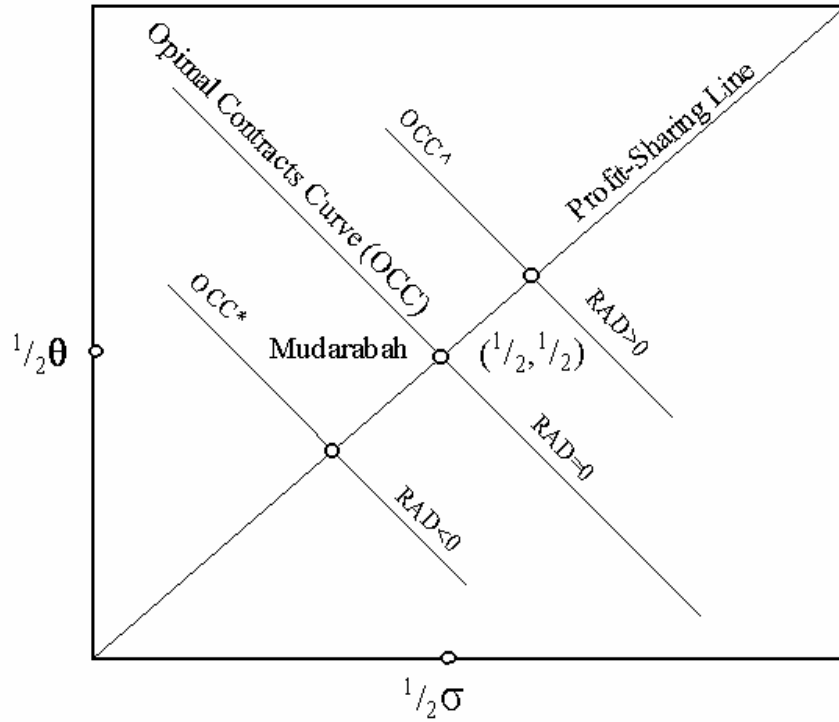
To represent the central point of the model (  $\frac{1}{2}$ ,  $\frac{1}{2}$  ) we shall substitute the fixed value  $\beta = \frac{1}{2}$  in [18] to get:

$$(c + 2a_2 \alpha \theta) / (c + 2b_2 (1-\alpha)\theta) = a_2 / b_2$$

The income ratio  $\alpha$  then turns out to be:

$$\begin{aligned} \alpha &= \frac{1}{2} + c(a_1 - b_2) / 4 a_2 b_2 \theta \\ &= \frac{1}{2} + k (\text{RAD} / \theta), \end{aligned} \quad [19]$$

Figure 2: Optimal contract curve for various risk-aversion differentials



Interestingly,  $\alpha$  is directly expressible in terms of a  $\theta$ -weighted RAD, apart from a positive constant  $k = c/4 a_2 b_2$ . It clearly affirms the previous finding that the equal income shares  $\alpha = 1/2$  corresponds to zero RAD. It also shows that for a large value of the total expected return  $\theta$  the weighted  $RAD/\theta$  will converge to zero, and hence  $\alpha = 1/2$  will tend to be a good approximation. That is, for large expected profits, the *muḍārabah* profit-sharing ratio will tend to be an equal shares ratio.

Otherwise, for cases where  $RAD/\theta$  is significantly different from zero, the relative position of the optimal contracting curve can then be shown with reference to figure (2) above. The main findings are as follows:

1. Party A closer to risk neutrality ( $RAD > 0$ ). Here  $\alpha > 1/2$ . As expected, the optimal contracting curve will fall to the right of the point  $(1/2, 1/2)$ .
2. Party B closer to risk neutrality ( $RAD < 0$ ). Here  $\alpha < 1/2$ . Hence, the optimal contracting curve will fall to the left of  $(1/2, 1/2)$ .

The figure also shows how the optimal profit-sharing ratio of *muḍārabah* financing is affected by the RAD. For the  $RAD = 0$ , the *muḍārabah* profit-sharing ratio is  $\alpha = 1/2$ . However, the ratio rises above  $1/2$  where  $RAD > 0$ , or drops down below  $1/2$  where  $RAD < 0$ .

### 5.3 How Likely is the Break-even Theory of *muḍārabah*

The question remains: how to decide on an optimal profit distribution ratio  $\alpha_0$  for *muḍārabah* financing? As it appears, this theoretical question becomes quite complicated by the unpredictable nature of an unobservable RAD. In principle, there are at least two reasons to believe that  $\alpha_0 = 1/2$  is the centre of gravity for the probability distribution of RAD, and any given  $\theta$  under the assumed competitive conditions. First, the generation of either positive or negative RAD will depend upon the nature of the matching process of the two-party contract within the financial market. If we assume a random matching process, then RAD will be a zero expectation random variable, resulting in:

$$E(\alpha) = 1/2 + k(E(RAD)/\theta) = 1/2 \quad [20]$$

In this respect, random matching will most likely neutralize the wealth effect which may account for the possibility of having different risk-aversion rates.

Second, the RAD may not be significantly different among contracting parties in actual practise. After all, risk-aversion is an embodiment of bounded concave utility function for money. The specification of money utility function through the cardinal expected utility approach makes it more tenable for making sensible inter-personal assumptions than the case with consumer good utility functions. The wealth effect is often cited as a decisive factor in the inter-person comparison of risk-aversion rates. Otherwise, there are hardly any grounds for remarkable pure taste differences in money to account for manifestly different utility functions for money.

### 5.4 The Pure Borrowing Model

As expected, the assumption of informational efficiency has to be abandoned if a neoclassical type of pure borrowing model is to emerge. The mathematical demonstration of this property is given at the Appendix. The emergence of pure borrowing/lending is, therefore, tied up with a problem of informational asymmetry

in the ex-ante sense. The neoclassical theory departs from the assumption that the entrepreneur (i.e. the manager B in the current model) is profit maximizing. In the final analysis, it is an information advantage which enables Party B (the entrepreneur) to maximize profit through borrowing at a relatively low interest rate, rather than sharing at the given sharing ratio  $\alpha_0$ .

In this sense, the existence of an interest based capital market reflects a fundamental property of informational asymmetry between borrower and lender as regards the risk return properties of economic prospects. The crux of the matter is that profit sharing entails disclosure of information among the parties involved, but profit maximizing agents would rather capitalize on their ex-ante relative information advantages in the fixed priced service markets. Apparently, it is the profit maximization incentive on the part of capital demanders that accounts for the institutional structuring of a non-sharing price mechanism where capital and labour services are sold at fixed prices. As discussed by Tag El-Din (2002), the neoclassical profit maximizing entrepreneur is perhaps the best model of demand-following labour and capital markets(13).

## 6. Concluding Remarks

The main implication of the above analysis is that the risk sharing structure of a manager/financier contract cannot be viewed independently of its income sharing ratio. In particular, the ethical appeal of *muḍārabah* financing relates, not only to a fair risk-sharing property, but also to a significant tendency towards a fair income ratio. In particular, the *muḍārabah* profit-sharing ratio cannot be freely manipulated in a discretionary monetary policy, without consideration to possible adverse effects on the risk-sharing structure of *muḍārabah* – see Siddiqi (1983) and Uzair (1982) for the recommended use of profit-sharing ratio in a monetary policy set-up.

Obviously, this finding may not hold strictly if the assumption of expected utility maximization is relaxed and an altruistic behaviour is assumed. At any rate, the finding that the *muḍārabah* contract is analytically prone towards a fair income ratio provides a more appealing interpretation to the concept of economic justice in *muḍārabah* than the one based on altruistic behaviour. It may happen that one of the two parties voluntarily accepts a very small share in a *muḍārabah* profit, or no share at all. As mentioned previously, the idea of *ibdaa'* in Islamic jurisprudence is a case in point where all profit is donated by the *muḍārib* to the financing party (*rubb al-māl*). But such altruism cannot be taken as the general rule at the market level. From competitive market perspectives, a very low income ratio for 'manager' vis-a-vis 'financier' is more indicative of manager's weak bargaining position than one of benevolence.

Hence, the finding that *muḍārabah* is a competitive breakeven point in terms of both income ratio and risk-sharing structure, is ethically sound. The prevalent trend

is to emphasize the ethical appeal of *muḍārabah* financing with reference to its risk-sharing provision that does not exist in the rival fixed interest financing mode. However, little attention is paid to meaningful differences in the income ratio, although income is the primary positive utility value. This has resulted in more attention being placed on the risk-sharing structure of Islamic modes than on their income sharing structures.

An important prediction of the above analysis is that: economically weaker parties are at a great disadvantage, as they are left with smaller income shares and larger risk shares. This explains the emergence of the *muḍārabah* contract as break-even point where the two competitive market forces are fairly balanced. The model predicts that a relatively strong market position for capital owners is non-conducive to an Islamic interest-free system, since it would generate a sufficiently high income ratio for capital owners,  $\alpha_0 > \frac{1}{2}$ , and hence induce partial borrowing at a risk-free interest. In this sense, the emergence of borrowing is linked with a relatively weak bargaining position of manager.

However, it makes little sense to argue that borrowing managers are weak parties in the capital market! Admittedly, these predictions are highly sensitive to the assumption of an informational efficient market. In particular, the neoclassical entrepreneur, who purely borrows capital at fixed interest, cannot be considered the weak party of the contract. On one hand, the pure borrowing model cannot exist in an informational efficient environment. On the other hand, the entrepreneur operates in an informational inefficient market where information is recognizable as a source of market power (Tag El-Din, 2002). The formal conditions for the emergence of pure risk-free lending within an informational inefficient market, are provided in the appendix. As it appears, the possible command over scarce market information may tilt the balance of economic power towards otherwise weak parties. It tends to justify, not only a potentially large income share for profit-maximizing entrepreneur, but also a purely risk-free income to capital owners.

To sum up, the conditions conducive for *muḍārabah* require fair competitive balance of market powers, not only in terms of relative bargaining positions, but also in terms of the information resource.



## Appendix

### How Pure Borrowing Arises?

We shall establish the conditions which give rise to pure borrowing by departing from any optimal risk-sharing contract  $(\alpha_0\theta, \beta_0\sigma)$ . Without loss of generality, the latter can be considered an optimal *muḍārabah* contract. As it has been shown above,  $\alpha_0 = \beta_0$  in *muḍārabah*. At this position there exists a lending risk-free rate  $r^*$  which makes Party A indifferent between *muḍārabah* and pure lending. Correspondingly, for Party B there exists a risk-free borrowing rate  $r^\wedge$  which makes him indifferent between *muḍārabah* and borrowing.

Yet, the convexity of the indifference curves implies the fundamental property that:  $r^\wedge < r^*$  as shown in figure (3) below. This property is very vital to the subsequent analysis. At the optimal *muḍārabah* contract, Party B would strictly prefer pure borrowing at a sufficiently low interest rate,  $r_0$  where:

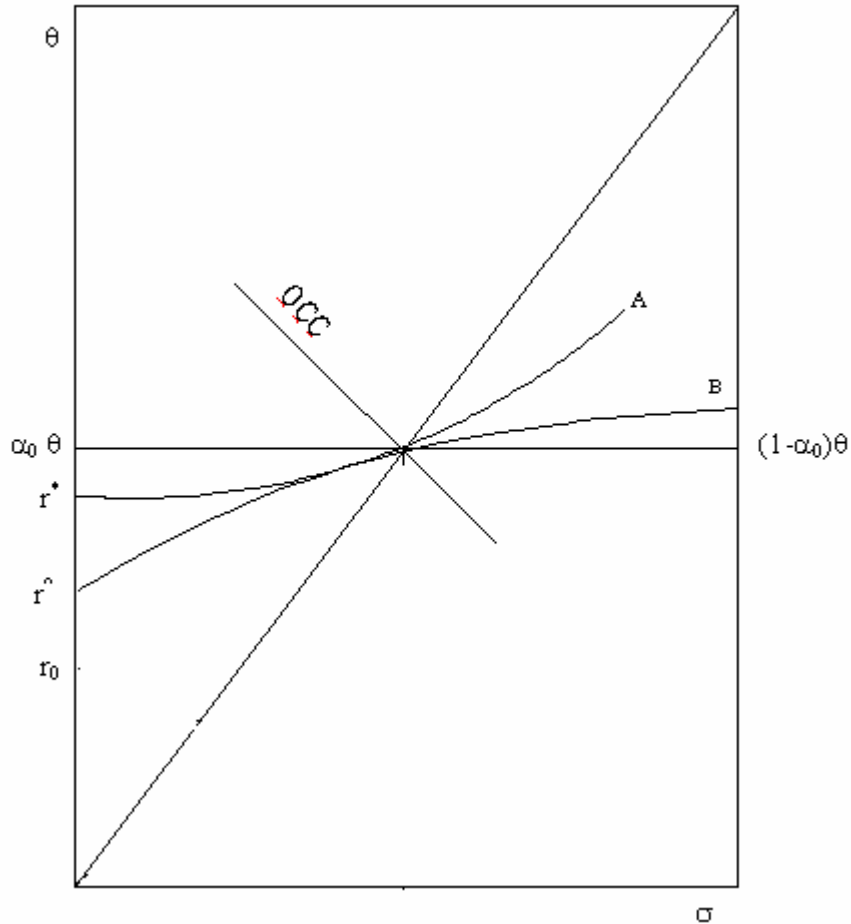
$$U_B(\theta - r_0, \sigma) > U_B((1-\alpha_0)\theta, (1-\beta_0)\sigma) \text{ for } r_0 < r^\wedge \quad [A1]$$

But this would make Party A worse off, given that  $r^\wedge < r^*$ . Alternatively, Party A will strictly prefer pure lending at a sufficiently high interest rate,  $r^\bullet$ , where:

$$U_A(r^\bullet, 0) > U_A(\alpha_0\theta, \beta_0\sigma), \text{ for any } r^\bullet > r^*, \quad [A2]$$

But, again due to  $r^\wedge < r^* < r^\bullet$  this will make Party B worse off. Indeed, it is the failure to reconcile conditions [A1] and [A2] which yields *muḍārabah* as the optimal solution of the financial choice problem. Therefore, pure borrowing/lending will only prevail if the above two conditions are reconciled. This situation is depicted in figure (3) below:

Figure 3: The starting point, an optimal muḍārabah position



To reconcile the above two conditions [A1] and [A2], so that the pure borrowing contract appeals for both parties, it is important to create a situation where

$$r^{\wedge} > r^* \text{ so that } r^{\bullet} < r^* < r^{\wedge}$$

The only conceivable situation to satisfy this criterion is where the RRSM is not the same for both parties. In other words, information about  $\theta$  and  $\sigma$  is not the same for both parties. Since Party B is a profit-maximizing entrepreneur working at the fore front of information-making activity, Party A must be the misinformed party. Accordingly, Party B has an informational advantage over Party A.

In this case, we will re-write condition A1] above in terms of a modified parameter  $(\theta', \sigma)$  such that  $\theta' > \theta$  where  $\theta'$  is assumed to be the true expectation of total return held by the entrepreneur, B.

Without loss of generality, the presentation can be simplified by imposing the equality  $\theta = \alpha_0 \theta'$ . In other words, what Party B expects as the profit share of Party A in a *muḍārabah* contract  $(\alpha_0 \theta')$ , is assumed equal to what the ‘misinformed’ Party B expects to be the total profit of the project ( $\theta$ ). Also, for simplicity, it is assumed that both parties agree on the same risk perception due to the fact that  $\sigma$  is the same. This results in two RRSM boxes: a larger one perceived by Party B and a shorter one perceived by the misinformed Party A. The profit share in *muḍārabah* perceived by party B is defined in the lower box at  $\alpha_0 \theta$  which indicates that Party A has a much lower return perception, than Party B and, hence, a low  $r^*$  as shown in the shorter RRSM of figure (4) below. The bigger RRSM will, therefore, be defined for Party B with a lower  $r^*$ , such that:

$$r^{\wedge} > r^*$$

This is shown in figure (4) below. Hence, depending on the extent to which  $\theta' < \theta$ , it will be possible to have a common  $r_0$  to match the two parties where:

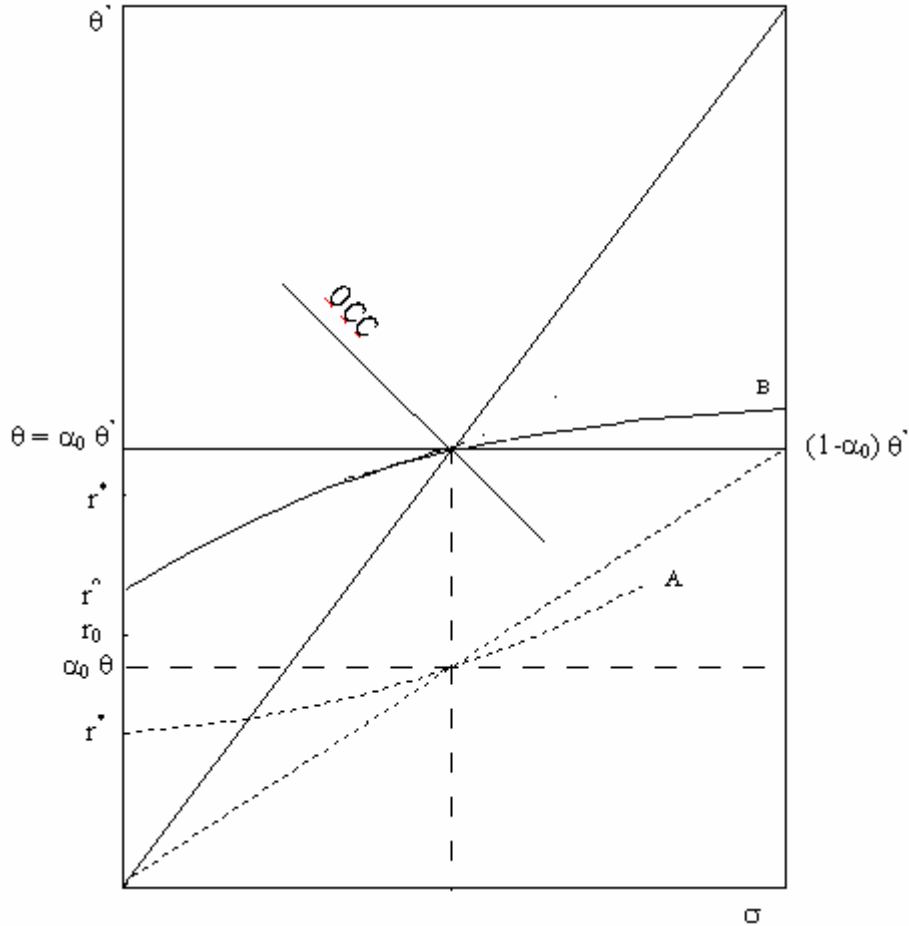
$$U_A(r_0, 0) > U_A(\alpha_0 \theta', \beta\sigma), \text{ and}$$

$$U_B(\theta - r_0, \sigma) > U_B((1-\alpha_0)\theta, (1-\beta)\sigma),$$

$$\text{for } r^* < r_0 < r^{\wedge}$$

[A3]

Figure 4: The rise of pure borrowing



The emergence of pure borrowing/lending is, therefore, tied up with a problem of informational asymmetry in the ex-ante sense. In the first place, there is an incentive for Party B to maximize profit through borrowing at a relatively low interest rate, rather than sharing at the given sharing ratio  $\alpha_0$ . The crux of the matter is that profit sharing entails disclosure of information among the parties involved, but profit maximizing agents would rather capitalize on their ex-ante relative information advantages in the fixed priced service markets. Apparently, it is the profit maximization incentive on the part of capital demanders that accounts for the institutional structuring of a non-sharing price mechanism where capital and labour services are sold at fixed prices. As discussed by Tag El-Din (2002), the neoclassical profit maximizing entrepreneur is perhaps the best model of demand-following labour and capital markets.

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